Let us assume the accuracy to be 3% for the hypothesis of shear stresses, therefore $\varepsilon = 0.03$. Let us present values of p calculated from (3.4) for some values of λ_1

$\lambda_1 = 0.05$	0.1	0.3	0.5	0.7	0.9	0.95
$p < 12$	3	$3\cdot10^{-1}$	$9\cdot10^{-2}$	$3\cdot10^{-2}$	$7\cdot10^{-3}$	$3.3.10^{-3}$

BIBLIOGRAPHY

1. Gusein-Zade, M. I., Construction of Bending Theory of Laminar Plates. Transactions of All-Union Confer. on Plates and Shells Theory, Baku. Moscow, "Nauka" 1966.
2. Gusein-Zade, M. I., On the derivation of a theory of bending of layered plates. PMM Vol. 32, №2, 1968.
3. Gol'denveizer, A. L., Derivation of an approximate theory of bending of a plate by the method of asymptotic integration of the equations of the theory of elasticity. PMM Vol. 26, №4, 1962.
4. Gol'denveizer, A. L. and Kolos, A. V., On the derivation of two-dimensional equations in the theory of thin elastic plates. PMM Vol. 29, №1, 1965.
5. Lur'e, A. I., On the theory of thick plates. PMM Vol. 6, №2-3, 1942.
6. Kurshin, L. M., Survey of research on the analysis of sandwich plates and shells. Sb. "Analysis of Three-dimensional Structures", Moscow, Gosstroiizdat, №7, 1962.
7. Aksentian, O. K. and Vorovich, I. I., The state of stress in a thin plate. PMM Vol. 27, №6, 1963.
8. Pinney, E., Ordinary Differential-Difference Equations. (Russian Translation), Moscow, IIL, 1961.

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ON THE STABILITY OF COMPRESSED BARS

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The problem of the stability of an incompressible elastic bar of variable stiffness, compressed along the axis, is considered. The validity of linearization is proved, and the equilibrium modes after buckling are investigated.

After reduction of the appropriate boundary value problem to an equation with a completely continuous operator, a theorem of Krasnosel'skii [1] on bifurcation can be applied. In utilizing this theorem the proof of the simplicity (or odd multiplicity) of the eigenvalue of the corresponding linearized problem is the principal difficulty.

The case of hinged supports of the bar ends was considered in [2]. In this case the linearized equation is of second order, and the simplicity of the eigenvalues results from the Sturm-Liouville theory. More complex support cases are considered herein (Sect. 2), and the spectrum of the differential equation $Au = \lambda Bu$ must be investigated, where A is a fourth order differential operator, B is of second order. This equation reduces to the equation $Nu = \lambda u$, where N is a second order differential operator. A problem with non-Sturm boundary conditions can occur for this reduction. In this case, the results of Kalafati on second order differential operators [3] can be used to prove the simplicity of the spectrum.

Let us note that the validity of linearization in the plate stability problem was given a foundation by the author of [4] (*).

The nature of the bifurcation of the even equilibrium modes of a symmetric bar is investigated in Sect. 3. It is shown by the Liapunov-Schmidt method that the membrane state of stress of the bar loses stability when the loading parameter becomes greater than the first eigenvalue of the linearized problem (Euler critical force). Two new equilibrium modes hence occur.

It has also been established that buckling is of Euler character: there are no other equilibrium modes, except the membrane mode, when the loading parameter is less than the Euler critical force.

The example (3.9) shows that odd equilibrium modes can occur in the elastic support case for subcritical values of the loading parameter.

1. Equilibrium equation and boundary conditions. Let us consider an incompressible elastic bar of variable cross section, clamped elastically in elastic supports. The bar is compressed by two horizontal forces λ applied to the ends. Let us introduce the following notation: E is the Young's modulus of the bar material, s the arclength of the bar measured from some fixed point, $J(s)$ the moment of inertia of the bar section at the point s. The function $y(s)$ is the deflection of the bar measured from the membrane equilibrium state.

The potential energy of a bent bar is

$$U = {}^1/_2 \int_{-1}^{1} \frac{EJy''^2}{1-y'^2} ds - \lambda \int_{-1}^{1} [1 - \sqrt{1-y'^2}]\, ds + {}^1/_2 \alpha_1 y^2(-1) + {}^1/_2 \alpha_2 y^2(1) + {}^1/_2 \gamma_1 \arcsin^2 y'(-1) + {}^1/_2 \gamma_2 \arcsin^2 y'(1) \qquad (1.1)$$

where $\alpha_1 > 0$ and $\alpha_2 > 0$ are the coefficients of elasticity of the supports, $\gamma_1 > 0$ and $\gamma_2 > 0$ the coefficients of elasticity of the clamping.

The condition of extremum of the energy $\delta U = 0$ results in the equilibrium equation

$$\frac{d}{ds} \frac{1}{\sqrt{1-y'^2}} \frac{d}{ds} \frac{EJy''}{\sqrt{1-y'^2}} = -\lambda \frac{d}{ds} \frac{y'}{\sqrt{1-y'^2}} \qquad (1.2)$$

and the boundary conditions

$$\frac{d}{ds} \frac{EJy''}{\sqrt{1-y'^2}} = -\alpha_1 y \sqrt{1-y'^2} - \lambda y'_{s=-1}, \quad \frac{d}{ds} \frac{EJy''}{\sqrt{1-y'^2}} = \alpha_2 y \sqrt{1-y'^2} - \lambda y' |_{s=1}$$

$$\frac{EJy''}{\sqrt{1-y'^2}} = \gamma_1 \arcsin y' |_{s=-1}, \qquad \frac{EJy''}{\sqrt{1-y'^2}} = -\gamma_2 \arcsin y' |_{s=1} \qquad (1.3)$$

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*) I. I. VOROVICH: Some Mathematical Questions of the Nonlinear Theory of Shells. Dissertation, Leningrad State University, 1958.

In the case of rigid clamping of both ends of the bar ($\alpha_1 = \alpha_2 = \gamma_1 = \gamma_2 = \infty$) the boundary conditions are

$$y(\mp 1) = y'(\mp 1) = 0 \tag{1.4}$$

If the left end of the bar is rigidly clamped, and the right end is hinge supported ($\alpha_1 = \gamma_1 = \alpha_2 = \infty,\ \gamma_2 = 0$), we have

$$y(\mp 1) = y'(-1) = y''(1) = 0 \tag{1.5}$$

The problem under consideration is conservative for any of the boundary conditions (1.3),(1.4),(1.5).

Let the bar be compressed by the follower forces λ applied to the ends, i.e. forces whose direction (at each instant) agrees with the tangent to the elastic line of the bar. Such a system is nonconservative. The boundary conditions hence are

$$\frac{d}{ds}\frac{EJy''}{\sqrt{1-y'^2}} = -\alpha_1 y\sqrt{1-y'^2}\,\Big|_{s=-1}, \qquad \frac{d}{ds}\frac{EJy''}{\sqrt{1-y'^2}} = \alpha_2 y\sqrt{1-y'^2}\,\Big|_{s=1}$$
$$\frac{EJy''}{\sqrt{1-y'^2}} = \gamma_1 \arcsin y'\,\Big|_{s=-1}, \qquad \frac{EJy''}{\sqrt{1-y'^2}} = -\gamma_2 \arcsin y'\,\Big|_{s=1} \tag{1.6}$$

2. On the bifurcation of the equilibrium of a bar of variable stiffness. 2.1°. Reduction to an operator equation. The nonlinear equilibrium equation (1.2) is a particular case of the equation

$$(EJy'')'' + \lambda y'' = f(s, y, y', y'', y''', \lambda) \tag{2.1}$$
$$\left(\lim_{t\to 0}\frac{f(s, ty, ty', ty'', ty''', \lambda)}{t} = 0\right)$$

where $f(s, y, y', y'', y''', \lambda)$ is a function of its arguments continuously differentiable for $|s| \leqslant 1$ and sufficiently small y, y', y'', y'''.

Inverting the linear part of the differential operator (1.2) for $\lambda = 0$, we reduce each of the boundary value problems (1.3)–(1.6) to an operator equation of the form $y = K(y, \lambda)$ with completely continuous operator in $C^{(4)}(-1, 1)$.

For example, the operator K in the cases (1.4),(1.5) is

$$(Ky)(s) = -\lambda\int_{-1}^{1} G(s,\xi)\,y''(\xi)\,d\xi + \int_{-1}^{1} G(s,\xi)\,f(\xi, y, y', y'', y''', \lambda)\,d\xi \tag{2.2}$$

where $G(s, \xi)$ is the Green's function for these boundary conditions. The Fréchet derivative of the operator K at the point $y = 0$ is a linear operator

$$(Gy)(s) = -\lambda\int_{-1}^{1} G(s,\xi)\,y''(\xi)\,d\xi \tag{2.3}$$

According to the theorem of Krasnosel'skii [1], each odd-multiple characteristic number of the operator G is a point of bifurcation of the operator K, where a continuous branch of the eigenvectors of the operator K corresponds to this bifurcation point.

2.2°. Spectrum of the linear problem and bifurcation. The eigenvalue problem for the operator (2.3) is evidently equivalent to the spectral problem for the linearized equation

$$(EJy'')'' = -\lambda y'' \tag{2.4}$$

with linearized boundary conditions. Conditions (1.4) and (1.5) hence remain unchanged, but conditions (1.3) and (1.6) are reduced to the following:

$$(EJy'')' = -\alpha_1 y - \lambda y' |_{s=-1}, \qquad (EJy'')' = \alpha_2 y - \lambda y' |_{s=1} \tag{2.5}$$
$$EJy'' = \gamma_1 y' |_{s=-1}, \qquad EJy'' = -\gamma_2 y' |_{s=1}$$

$$(EJy'')' = -\alpha_1 y |_{s=-1}, \qquad (EJy'')' = \alpha_2 y |_{s=1} \tag{2.6}$$
$$EJy'' = \gamma_1 y' |_{s=-1}, \qquad EJy'' = -\gamma_2 y' |_{s=1}$$

The existence of an infinite sequence of positive eigenvalues of Eq. (2.4) with the boundary conditions (2.5), (1.4), (1.5) results from known variational theorems [5].

Theorem 2.1. The problem (2.4), (1.5) has an infinite sequence of eigenvalues. All the eigenvalues are positive, simple, and are bifurcation points of the nonlinear operator K.

Proof. Integrating (2.4) twice we obtain

$$E\ Jy'' + \lambda y = \lambda c_1 s + \lambda c_2 \tag{2.7}$$

Making the substitution $y(s) = v(s) + c_1 s + c_2$ in the equations and boundary conditions, and eliminating the unknown constants c_1 and c_2, we obtain the equivalent Sturm-Liouville problem

$$EJv'' + \lambda v = 0, \quad v(-1) + 2v'(-1) = 0, \quad v(1) = 0 \tag{2.8}$$

The corresponding Green's function is oscillating (see [6]), and hence, all the eigenvalues are positive and simple.

2.3°. Let us utilize the results of Kalafati [3] in investigating the problem (2.4), (1.4). The second order differential equation

$$Ly = \lambda \rho(x) y$$
$$Ly = -d/dx\,(py') + qy, \quad p(x) > 0, \quad q(x) > 0 \quad (a \leqslant x \leqslant b)$$

with the boundary conditions

$$\alpha_{11}\, y(a) + \alpha_{12}\, y'(a) + \beta_{11}\, y(b) + \beta_{12}\, y'(b) = 0$$
$$\alpha_{21}\, y(a) + \alpha_{22}\, y'(a) + \beta_{21}\, y(b) + \beta_{22}\, y'(b) = 0 \tag{2.9}$$

is considered in this work.

The following sufficient conditions are given in [3] for the Green's function of the operator L to be an even or odd K-kernel.

The matrix of the coefficients of conditions (2.9) is

$$\left\| \begin{matrix} \alpha_{11} & \alpha_{12} & \beta_{11} & \beta_{12} \\ \alpha_{21} & \alpha_{22} & \beta_{21} & \beta_{22} \end{matrix} \right\| \tag{2.10}$$

Let the symbol $\{i, k\}$ be the second order determinant of the matrix (2.10) into whose composition columns with the numbers i and k $(i < k)$ enter. If the conditions

$$\{2,4\} \neq 0, \qquad \{2,4\} \cdot \{1,2\} > 0 \tag{2.11}$$

are satisfied, the Green's function of the operator L is an odd K-kernel. If

$$\{2,4\} \neq 0, \qquad \{2,4\} \cdot \{1,2\} < 0 \tag{2.12}$$

the Green's function is an even K-kernel.

Theorem 2.2. The problem (2.4), (1.4) has an infinite sequence of eigenvalues. All the eigenvalues are not more than double: $0 < \lambda_0 \leqslant \lambda_1 < \lambda_2 \leqslant \lambda_3 < \ldots$; the appropriate eigenvalue has no more than $n + 3$ zeros for even n, and no more than $n + 2$ zeros for odd n in the interval $(-1, 1)$.

Proof. As in the proof of Theorem 2.1, let us turn to the consideration of the second order problem

$$EJv'' + \lambda v = 0,\ v(1) - v(-1) = 2v'(1),\ v'(1) - v'(-1) = 0 \qquad (2.13)$$

The boundary conditions of the problem (2.13) are non-Sturm conditions. It follows from (2.12) that the Green's function of the operator EJv'' is an even K-kernel. Therefore, all the eigenvalues are not more than double

$$0 < \lambda_0 \leqslant \lambda_1 < \lambda_2 \leqslant \lambda_3 < \ldots$$

and hence the nth eigenfunction v_n has n or $n+1$ zeros for even n, and has n or $n-1$ zeros for odd n. Hence, by Rolle's theorem, we obtain an estimate of the number of zeros of the eigenfunction y_n.

2.4°. Let us consider a symmetric bar $(J(s) = J(-s))$ compressed by horizontal forces λ. Hence, $\alpha_1 = \alpha_2$ and $\gamma_1 = \gamma_2$ in the appropriate boundary conditions (1.2) and (2.5). Writing the function $f(s, y, y', y'', y''', \lambda)$ in Eqs. (1.2), it is easy to see that the operator $(Fy)(s) = f(s, y, y', y'', y''', \lambda)$ transforms the even function $y(s)$ into even, and the odd into odd. Let us investigate symmetric equilibrium modes of the bar. To do this let us consider the problem (2.4), (2.5) in the subspace of even functions.

Theorem 2.3. The problem (2.4), (2.5) has an infinite sequence of eigenvalues in the subspace of even functions. All the eigenvalues are positive, simple, and are bifurcation points of the operator K.

Proof. As in the preceding theorems, let us turn to the equivalent problem for $\lambda \neq 0$

$$EJv'' + \lambda v = 0, \qquad \gamma v'(1) = \lambda v(1), \qquad v'(0) = 0 \qquad (2.14)$$

Let us show that the problem (2.14) can be reduced to a loaded integral equation with monotonely increasing function $\sigma(\xi)$ and oscillating kernel. Since $\lambda_0 = 0$ is an eigenvalue of the problem (2.14), it is convenient to introduce the parameter μ by setting $\lambda = \mu - \varepsilon$. The problem (2.14) then becomes

$$Bv \equiv EJv'' - \varepsilon v = \mu v, \quad \gamma v'(1) + \varepsilon v(1) = \mu v(1), \quad v'(0) = 0 \quad (2.15)$$

The Green's function of the boundary value problem (2.15) is

$$G(s, \xi) = \begin{cases} v_1(s)\, v_2(\xi) & (0 \leqslant s \leqslant \xi \leqslant 1) \\ v_1(\xi)\, v_2(s) & (0 \leqslant \xi \leqslant s \leqslant 1) \end{cases} \qquad (2.16)$$

The functions v_1 and v_2 satisfy the conditions

$$Bv_1 = Bv_2 = 0,\ \gamma v_1'(1) + \varepsilon v_1'(1) = v_2'(0) = 0,\ v_1 v_2' - v_1' v_2 = 1$$

The solution of the problem (2.15) is

$$v(s) = \mu \int_0^1 G(s, \xi)\, v(\xi)\, d\xi - \mu v(1)\, \psi(s) \qquad (2.17)$$

where

$$\psi(s) = -G(s, 1)\, \gamma^{-1} EJ(1) \qquad (2.18)$$

Substituting (2.18) into (2.17), we obtain

$$v(s) = \mu \int_0^1 G(s, \xi)\, v(\xi)\, d\xi + \mu G(s, 1)\, \gamma^{-1} EJ(1)\, v(1) =$$

$$= \mu \int_0^1 G(s, \xi)\, v(\xi)\, d\sigma(\xi), \qquad \sigma(\xi) = \begin{cases} \xi & (0 \leqslant \xi < 1) \\ 1 + E\gamma^{-1} J(1) & (\xi = 1) \end{cases}$$

The Green's function $G(s, \xi)$ is oscillating [6]. The assertions of the theorem result from the theory of charged integral equations with oscillating kernel [6].

Note. That the Green's function of the problems (2.8) and (2.15) is oscillatory permits the proof that the nth eigenfunction of the problems (2.4), (1.5) and (2.4), (2.5) has no more than n zeros in the interval $(-1, 1)$.

The relation

$$y_n(s) = \lambda^2 \int_0^1 G(s, \xi) \frac{v_n(\xi)}{EJ(\xi)} d\xi \tag{2.19}$$

follows from (2.4).

According to the earlier proof, the function $v_n(\xi)$ has n zeros in the interval $(-1, 1)$. According to results of Krein [6], it follows from (2.19) that $y_n(s)$ has no more than n zeros in the interval $(-1, 1)$.

The problem (2.4), (1.4) can be examined separately in subspaces of even and odd functions. Theorems analogous to Theorem 2.3 are obtained in such an investigation of the symmetric and antisymmetric equilibrium modes of a bar.

3. Bifurcation. Theorem 3.1. In the case of the support conditions (1.3) and (1.4), no equilibrium mode different from the membrane mode, exists for a symmetric bar in the class of even equilibrium modes if $\lambda \leqslant \lambda_0$ is the first eigenvalue of the linearized problem. When λ becomes greater than λ_0, the membrane solution loses stability and two new equilibrium modes are generated which are representable as a power series in the parameter $\varepsilon = \sqrt{\lambda - \lambda_0}$

$$y_{1,2}(s) = \mp \varepsilon c y_0 + O(\varepsilon^2) \tag{3.1}$$

where the constant c is positive and determined by (3.8).

Proof. Let us assume that an even nonzero solution of the problem (1.2), (1.3) exists for some λ. Let us show that then $\lambda > \lambda_0$. Multiplying (1.2) by the function

$$z(s) = \int_{-1}^{s} y' \sqrt{1 - y'^2}\, dt$$

and integrating with respect to s between -1 and 1, we obtain

$$\lambda = [J^{(1)} + 2\gamma y'(1) \arcsin y'(1)] / J^{(2)} = J_1(y) \tag{3.2}$$

$$J^{(1)} = \int_{-1}^{1} \frac{EJy''}{\sqrt{1 - y'^2}} ds, \qquad J^{(2)} = \int_{-1}^{1} y'^2 ds$$

On the other hand, the eigenvalue λ_0 can be determined by means of the variational principle

$$\lambda_0 = \min J_2(y), \quad J_2(y) = \frac{1}{J^{(2)}} \Big(\int_{-1}^{1} EJy''^2 ds + 2\gamma y'^2(1) \Big) \tag{3.3}$$

where the minimum is taken over the set of even smooth functions. It is easy to see that $J_1(y) > J_2(y)$ for all y. Hence, it follows from (3.2) and (3.3) that $\lambda > \lambda_0$. The proof is analogous in the case of conditions (1.4).

Let us apply the Liapunov-Schmidt method to investigate the bifurcation. Let us seek the solution in the form of the power series

$$y = \sum_{k=1}^{\infty} \varepsilon^k y_k, \qquad \varepsilon = \sqrt{\lambda - \lambda_0} \tag{3.4}$$

Substitution into Eq. (1.2) and the boundary conditions results in a chain of differential equations

$$Py_1 = (EJy_1'')'' + \lambda_0 y_1'' = 0, \; Py_2 = 0 \tag{3.5}$$

$$Py_3 = -\, {}^1/_2\lambda_0 y_1'' y_1'^2 - 3\,(EJy_1'')' y_1' y_1'' \; - EJy_1''\,(y_1' y_1'')' - y_1'' \tag{3.6}$$

. . . .

From (3.5) we obtain that $y_1 = cy_0$, where y_0 is the solution of the appropriate linearized problem, and c is an unknown constant which we determine from the condition of solvability of Eq. (3.6). In the case of (1.4), for example, we obtain

$$c^2 = -\frac{1}{r}\int_{-1}^{1} y_0 y_0'' ds \tag{3.7}$$

$$r = \int_{-1}^{1} [{}^1/_2\lambda_0 y_0'' y_0'^2 + 3\,(EJy_0'')'\, y'_0 y_0'' + EJy_0''\,(y_0' y_0'')']\; y_0 ds$$

Integrating by parts, and utilizing the boundary conditions, we obtain

$$c^2 = \frac{6}{\lambda_0}\Big(\int_{-1}^{1} y_0'^2 ds\Big)\Big(\int_{-1}^{1} y_0'^4 ds\Big)^{-1} > 0 \tag{3.8}$$

The convergence of the series (3.4) for sufficiently small ε and the existence of the equilibrium pair (3.1) now follow from (3.8) and the known results of the Liapunov-Schmidt method (see [7], say). The equality (3.8) is true even in the support case (1.3). Let us note that the requirement for evenness is essential. Solutions can exist in the case (1.3) for loadings less than the first critical number of the linearized problem. For example, if $\gamma_1 = \gamma_2 = 0$, $\alpha_1 = \alpha_2 = \alpha$, then for $\lambda > 0$ there exist the odd solutions

$$y\,(s) = \mp\, s\sqrt{1 - \lambda^2 / \alpha^2} \tag{3.9}$$

while the first critical number equals α.

It is easy to show that for a symmetric bar the nonconservative problem (1.2), (1.6) is reduced to the problem (1.2), (1.3) by the substitution

$$y\,(s) = v\,(s) + \frac{\lambda v'\,(1)}{\alpha\sqrt{1 - v'^2\,(1)}}\,, \qquad v\,(s) = y\,(s) - \frac{\lambda y'\,(1)}{\alpha\sqrt{1 - y'^2\,(1)}}$$

Therefore, here Theorems 2.3 and 3.1 are also true. However, in this case this does not exhaust the question of bar buckling since the vibrational instability is also possible in nonconservative systems. For example, this occurs when $\alpha_1 = \gamma_1 = \infty$, $\alpha_2 = \gamma_2 = 0$ (see [8]).

The question of the existence and stability of self-oscillating regimes merits separate investigation. It can turn out that both types of buckling can occur for some values of the parameters. In such a situation it is necessary to clarify to which of them the lesser critical loading corresponds.

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BIBLIOGRAPHY

1. Krasnosel'skii, M. A., Topological Methods in the Theory of Nonlinear Integral Equations. Moscow, Gostekhizdat, 1956.
2. Bakhtin, I. A. and Krasnosel'skii, M. A., On the problem of the longitudinal bending of a variable-stiffness bar. Dokl. Akad. Nauk SSSR Vol. 105, №4, 1955.
3. Kalafati, P. D., *K*-properties of the Green's function of linear second-order differential systems. Uch. Zap. Khar'kov Univ. Vol. 80, 1957, Zap. Mat. Otd. Fiz.-Mat. Fak. and Khar'kov Matem. Ob-va, Ser. 4, Vol. 25.
4. Vorovich, I. I., Some questions of shell stability in the large. Dokl. Akad. Nauk SSSR Vol. 122, №1, 1958.
5. Mikhlin, S. G., Variational Methods in Mathematical Physics. Moscow, Gostekhizdat, 1957.
6. Gantmakher, F. R. and Krein, M. G., Oscillating Matrices and Small Oscillations of Mechanical Systems. Moscow-Leningrad, Gostekhizdat, 1941.
7. Vainberg, M. M. and Trenogin, V. A., Liapunov and Schmidt methods in the theory of nonlinear equations and their further development. Usp. Mat. Nauk Vol. 17, №2, 1962.
8. Bolotin, V. V., Nonconservative Problems of the Theory of Elastic Stability. Moscow, Fizmatgiz, 1961.

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ON SHOCKWAVE PROPAGATION IN AN ELASTIC SPACE WITH FINITE DEFORMATIONS

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The influence of finiteness of the deformations and of the convective terms, in determining the medium velocity in terms of the displacements, on shockwave propagation in a three-dimensional elastic medium is investigated. The Almansi tensor [1] is utilized as the finite strain tensor. It is found that the quantity of shocks and their properties depend strongly on the deformations of the medium ahead of the surface of strong discontinuity, and on whether or not nonlinear terms in the rheological equations are taken into account. Thus, propagation of three different shocks is possible in the case of small deformation when these equations are written exactly. The particular case when the medium is in the undeformed state ahead of the shock is singular: all the qualitative results agree with the results of the analogous linear problem. Expressions for the shock velocities are obtained explicitly in particular cases.

1. Let us write the connection between the stress tensor σ_{ij} and the Almansi finite strain tensor e_{ij} as

$$\sigma_{ij} = \lambda e_{kk}\delta_{ij} + 2\mu e_{ij}, \quad e_{ij} = {}^1/_2\,(u_{i,j} + u_{j,i} - \alpha u_{k,i}u_{k,j}) \qquad (1.1)$$

where λ and μ are the Lamé coefficients, u_i the displacements of the medium particles.